An Iterative Quantum Approach for Transformation Estimation from Point Sets

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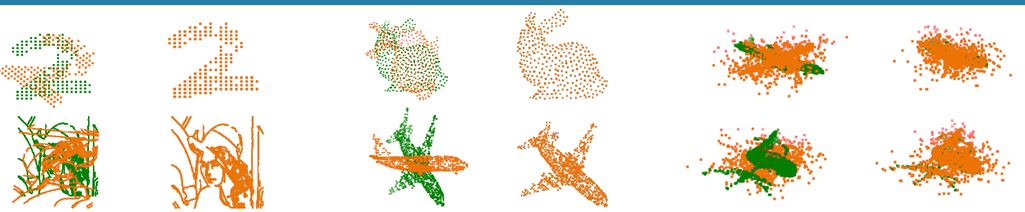
Overview

We propose an iterative method for **estimating rigid transformations** from point sets using **adiabatic quantum computation**.

$\check{\mathcal{X}} = \{\check{x}_i\}_{i=1}^N$ Reference point set (mass-centered),
$\check{\mathcal{Y}} = \{\check{y}_i\}_{i=1}^N$ Template point set (mass-centered),
$R \in SO(D)$ Rotation matrix that maps $\check{\mathcal{Y}}$ to $\check{\mathcal{X}}$ (has $P := D(D-1)/2$ degrees of freedom).

- Runs on an industrial adiabatic quantum computer – **D-Wave**.
- Recovers R to an **arbitrary precision** using a **user manageable number of qubits**.



(a) 2D. (b) 3D. (c) Noisy data.

Figure: Results of our iterative quantum approach for transformation estimation (IQT) from point sets. **Green:** reference points. **Orange:** template points. **Left:** Initial misalignments. **Right:** Registration results. Our method performs well on several 2D and 3D datasets and robustly aligns the points even in the presence of noise.

Objective

Task. Contruct for the least-squares formulation of the rigid point sets registration problem, cf. [1, 2]

$$\min_{R\in SO(D)} \sum_{i=1}^{N} \|\check{x}_i - R\check{y}_i\|_2^2 \qquad \text{(RPR)}$$

a corresponding QUBO formulation that can be executed on a current adiabatic quantum computer.

QUBO (quadratic unconstrained binary optimization). Combinatorial optimization problem in form of

$$\min_{q\in\{0,1\}^n} q^\top W q + c^\top q, \qquad (1)$$

with coupling matrix W and vektor of biases c that an adiabatic quantum computer can potentially solve.

Difficulties.

- Eq. (RPR) is not a binary problem since R is a real matrix.
- Eq. (RPR) is not an unconstrained problem since $R \in SO(D) \implies R^\top R \overset{!}{=} I$.

Adiabatic Quantum Computing

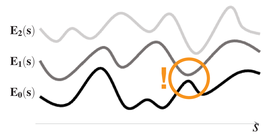
Principle. Construct two Hamiltonians, an inital hamiltonian H_0 with ground state $|E_0(0)\rangle = |+\rangle^{\otimes n}$ easy to prepare and a final hamiltonian H_1 with ground state $|E_0(1)\rangle$ encoding the solution of the QUBO problem:

$$H_0 = -\mathcal{A}\left(\sum_{k=1}^{n}\sigma_k^x\right), \quad \text{and} \quad H_1 = -\mathcal{B}\left(\sum_{k,l=1}^{n} W_{kl}\sigma_k^z\otimes\sigma_l^z + \sum_{k=1}^{n} c_k\sigma_k^z\right). \qquad (2)$$

For the time $s \in [0,1]$, let the time-dependent system Hamiltonian adiabatically evolve from H_0 to H_1 as

$$H(s) = (1-s)H_0 + sH_1. \qquad (3)$$

Adiabatic Theorem and Evolution.



If s varies slowly enough, the adiabatic theorem states that the evolution of the system governed by the Schrödinger equation

$$i\hbar\frac{\partial\,|\psi(s)\rangle}{T\partial s} = H(s)\,|\psi(s)\rangle\,, \qquad (4)$$

will instantaneously keep the state $|\psi(s)\rangle$ into the ground state $|E_0(s)\rangle$ of $H(s)$, which is at $s=1$ the solution of the QUBO [4].

Previous work

V. Golyanik and C. Theobalt

Quantum Approach (QA, [3]). Pick a finite set of basis matrices and approximate R with a binary linear combination. In 2D (3D similar) and using the matrix exponential for rotation matrices, approximate

$$R = e^{\theta S} = \cos(\theta)I + \sin(\theta)S \quad \text{with} \quad I = \begin{pmatrix}1&0\\0&1\end{pmatrix} \quad \text{and} \quad S = \begin{pmatrix}0&-1\\1&0\end{pmatrix}, \qquad (5)$$

as $R = \sum_k q_k Q_k$ using binary optimization variables $q_k \in \{0,1\}$ and selected basis matrices

$$Q_k \quad \in \quad \{\omega C \in \mathbb{R}^{2\times 2}, \quad \forall \omega \in B_1 := \{0.5, 0.2, 0.1, 0.1, 0.05\}, \quad \forall C \in B_2 := \{I, -I, S, -S\}\}.$$

Figure: Rigid point sets registration using QA [3]. **Left:** Initial misalignments. **Right:** Results of the registration.

New approach: Iterative Quantum Transformation Estimation

Idea. Use a flexible K-bits discretization of $SO(D)$ to approximate the rotation matrices with refinement, optimize over the parameters of R to enforce orthogonality and iterate the process to improve accuracy.

In 2D, parameter $\theta \in \mathbb{R}$ encodes rotation angle:

$$R = e^{M(\theta)} = \cos(\theta)I + \sin(\theta)S, \quad \text{with} \quad M(\theta) = \theta S = \theta\cdot\begin{pmatrix}0&-1\\1&0\end{pmatrix}.$$

In 3D, parameter $v = (v_1, v_2, v_3)^\top \in \mathbb{R}^3$ encodes rotation angle $\theta = \|v\|_2$ and rotation axis $x = v/\|v\|_2$:

$$R = e^{M(v)} = I + \frac{\sin\theta}{\theta}M(v) + \frac{1-\cos\theta}{\theta^2}M^2(v), \quad \text{with} \quad M(v) = \theta\cdot\begin{pmatrix}0&-x_3&x_2\\x_3&0&-x_1\\-x_2&x_1&0\end{pmatrix},$$

Until accuracy is satisfactory, iterate – (i) linearize, (ii) discretize , (iii) optimize:

(i) Linearize R around curent parameter $z_c := \theta_c$ (2D) resp. v_c (3D) using first order Taylor expansion.

(ii) Discretize parameter $z := \theta$ (2D) resp. v (3D) $\in [z_c - \Delta, z_c + \Delta]$ using the K-bits representation

$$z := z_c + Uq \quad \text{where with} \quad D_k = \text{diag}(2^k) \qquad (6)$$
$$U = \frac{2\Delta}{2^K}\left(D_0\, D_1 \ldots D_{K-1}\right) \in \mathbb{R}^{P,PK} \quad \text{and} \quad q = \left(q_0\, q_1 \cdots q_{K-1}\right)^\top \in \{0,1\}^{KP}.$$

(iii) For any of the points $\check{y}_i$ in (RPR) and with R_c being the constant term of R, we find [2] in 2D and in 3D

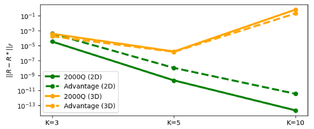
$$R\check{y}_i \approx R_c\check{y}_i + R_i U q. \qquad (7)$$

Crucially, q appears linearly in Eq. (7), so (RPR) can be approximated by

$$\min_{q\in\{0,1\}^{KP}} q^\top W q + c^\top q, \qquad \text{(QUBO)}$$
$$W = U^\top\left(\sum_{i=1}^{N} R_i^\top R_i\right)U \quad \text{and} \quad c = 2U^\top\sum_{i=1}^{N} R_i^\top(R_c\check{y}_i - \check{x}_i). \qquad (8)$$

Experiments and Results

Ablation Study on K. The method performs well for $K = 10$ in 2D and $K = 5$ in 3D.



Larger values of K result, due to noise, in chain breackage that decrease the results.

	QA [3]	IQT [Ours]
Optimization Variable	Matrix R	Parameter θ resp. v
Optimization Scheme	Fixed → fixed accuracy	iterative → **flexible accuracy**
R orthogonal	×	✓
Number of qubits	$\lvert B_1\rvert\cdot\lvert B_2\rvert$ → **21 in 2D, 81 in 3D**	$K\cdot P$ → **10 in 2D, 15 in 3D.**

Table: A comparative analysis of our optimization method with QA [3].

(a) 2D (b) 3D

Figure: Embedding of our derived QUBOs for Eq. (RPR) on the D-Wave $2,000Q$ quantum annealer [5]. We use ~ 4.5 times more physical than logical qubits and perform 100 reads of $20\mu s$ per iteration.

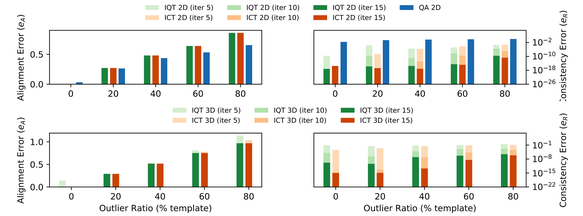


Figure: **Left:** Alignment error $e_A := \|\check{\mathcal{X}} - R\check{\mathcal{Y}}\|_F / \|\check{\mathcal{X}}\|_F$ and **right:** consistency error $e_R := \|I - R^\top R\|_F$ (note log. scale) of IQT **[ours]** and its classical ICT counterpart **[ours]** against outliers. Our methods, compared to QA **[3]**, robustly handle outliers while producing orthogonal matrices.

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