A Universal Quantum Algorithm for Weighted Maximum Cut and Ising Problems

MIC Institute of Mathematics and Image Computing

Natacha Kuete Meli, Florian Mannel, Jan Lellmann

{meli, mannel, lellmann}@mic.uni-luebeck.de

Goal: Prepare the ground-state of the $2^n \times 2^n$ Ising Hamiltonian

$$\mathbf{C} = \sum_{i=1}^{n} \mathcal{C}_{ii}\mathbf{Z}_i + \sum_{1\le i<j\le n} \mathcal{C}_{ij}\mathbf{Z}_i\mathbf{Z}_j.$$

Or equivalently, minimize$_q$ $\langle q|\mathbf{C}|q\rangle$ s.t. $q \in \{0,1\}^n$.

Applications in computer vision

- Maxcut for image segmentation[1]
- Ising for shape matching[2], point sets registration[3], etc...

Challenges

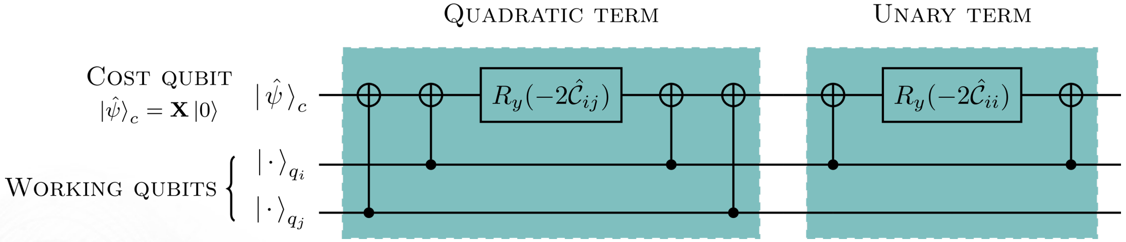
- Combinatorial problem
- Hard to solve with conventional computers

Method[4]: Tackle the problem on a universal quantum device

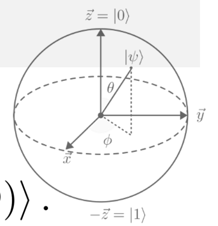
Embed $\mathbf{C}$ into the $2^{1+n} \times 2^{1+n}$ unitary and Hermetian operator

$$\mathbf{U} := \begin{bmatrix} \sin(\hat{\mathbf{C}}) & \cos(\hat{\mathbf{C}}) \\ \cos(\hat{\mathbf{C}}) & -\sin(\hat{\mathbf{C}}) \end{bmatrix} \text{ for } \hat{\mathbf{C}} := \frac{\mathbf{C}}{K},\ K \in \mathbb{R} \text{ such that } \hat{\mathbf{C}} \in [\tfrac{\pi}{2}, \tfrac{\pi}{2}]^{2^n}.$$

Implement $\mathbf{U}$ using the following circuit:



Then: minimize$_q$ $\langle q|\mathbf{C}|q\rangle$ $\Leftrightarrow$ minimize$_q$ $\langle 0,q|\mathbf{U}|0,q\rangle$.

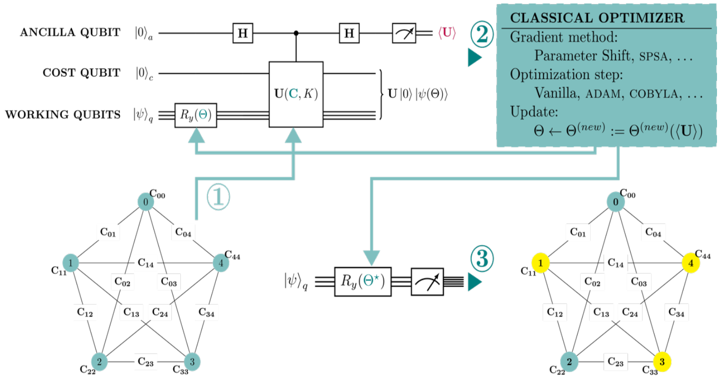


Optimization workflow

In an hybrid variational quantum circuit regime,

$$\underset{\Theta\in\mathbb{R}^n}{\text{minimize}} \quad \mathcal{L}(\Theta), \quad \mathcal{L}(\Theta) = \langle 0,\psi(\Theta)|\mathbf{U}|0,\psi(\Theta)\rangle.$$

Evaluate the function on a quantum machine and optimize classically:



Proposed algorithm

Input: Graph $\mathcal{G} = (\mathcal{S}, \mathcal{E}, \mathcal{C})$

Initialize: Parameter vector $\Theta \in \mathbb{R}^n$

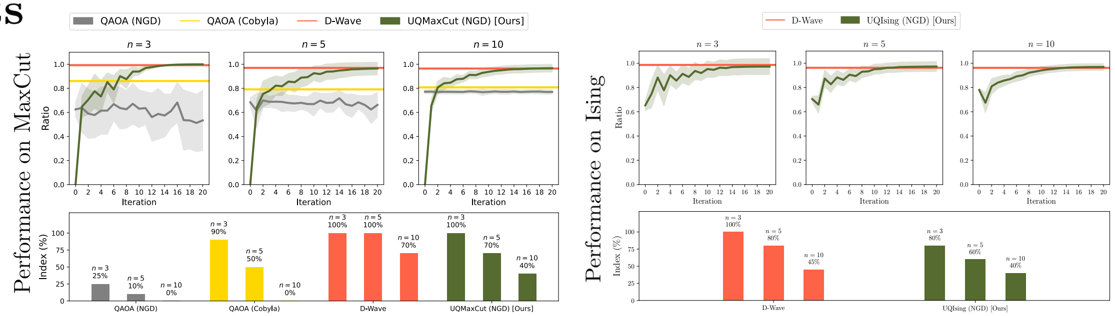
Repeat: (1) Prepare $|\psi(\Theta)\rangle$ and perform $\mathbf{U}\,|0,\psi(\theta)\rangle$

(2) Measure $\langle\mathbf{U}\rangle$ and update $\Theta \leftarrow \Theta^{(new)}(\langle\mathbf{U}\rangle)$

And finally:

(3) Sample $|\psi(\Theta^\star)\rangle$ and output the most frequent basis-state $q^\star$

Results



References

1. Kolmogorov et al.: What energy functions can be minimized via graph cuts? (IEEE TPAMI 2004)
2. Benkner, et al.: Q-match: Iterative shape matching via quantum annealing. (ICCV 2021)
3. Meli et al.: An iterative quantum approach for transformation estimation from point sets. (CVPR 2022)
4. Meli et al.: A universal quantum algorithm for weighted maximum cut and Ising problems. (In press @ Springer Quantum Information Processing)

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